Homework 6

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6.1 A simple harmonic one-dimensional oscillator has energy levels given by $E_n = (n + \frac{1}{2})\hbar\omega$, where ω is the characteristic (angular) frequency of the oscillator and where the quantum number n can assume the possible integral values $n = 0, 1, 2, ...$ Suppose that such an oscillator is in thermal contact with a heat reservoir at temperature T low enough so that $\frac{kT}{\hbar\omega} << 1$.
(a) Find the ratio of the probability of the oscillator being in the first excited state to the probability of its being in the ground state.
(b) Assuming that only the ground state and first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature T.
(c) Not in Reif. Assume $\frac{kT}{\hbar\omega} = 0.5$, meaning that the thermal energy is half of the harmonic oscillator energy spacing. Compute numerical answers for parts (a) and (b). For part (b) express the answer in units of $\hbar\omega$.

6.2 Consider again the system of Problem 3.2, i.e. N weakly interacting particles, each of spin $\frac{1}{2}$ and magnetic moment μ, located in an external field H. Suppose that this system is in thermal contact with a heat reservoir at the absolute tempreature T. Calculate its mean energy $\bar{E}$ as a function of T and H. Compare the result with the answer to Problem 3.2a.

6.6 A system consists of N weakly interacting particles, each of which can be in either of two states with respective energies ϵ_1 and ϵ_2, where $\epsilon_1 < \epsilon_2$.
(a) Without explicit calculation, make a qualitative plot of the mean energy $\bar{E}$ of the system as a function of its temperature T. What is $\bar{E}$ in the limit of very low and very high temperature? Roughly near what temperature does $\bar{E}$ change from its low to its high temperature limiting values?
(b) Using the result of (a), make a qualitative plot of the heat capacity C_V (at constant volume) as a function of the temperature T.
(c) Calculate explicitly the mean energy $\bar{E}(T)$ of this system. Verify your predictions for low T, high T and the transition temperature.
(d) Calculate the heat capacity $C_V(T)$ of this system. Hints: define $\bar{\epsilon} = \frac{\epsilon_1+\epsilon_2}{2}$ and $\Delta\epsilon = \epsilon_1 - \epsilon_2$ and use these instead of ϵ_1 and ϵ_2; also, $\frac{d}{dx}\tanh(x) = \mathrm{sech}^2(x)$.

4. Consider a quantum harmonic oscillator, for which the energy levels are $E = \hbar\omega(n + \frac{1}{2})$.
(a) Compute its canonical partition function, Z, simplifying the result to a simple closed-form expression.
(b) Compute its Helmholtz free energy, F.
(c) Compute the mean energy, $\bar{E}$.
(d) Compute the mean energy if $kT << \hbar\omega$ and if $kT >> \hbar\omega$.
(e) What is the mean energy level number, $\bar{n}$, for the vibration of hydrogen molecules (H_2) on the surface of the sun? (H_2 vibrational frequency is $1.25 \cdot 10^{14}$ s^{-1}, Planck's constant is $6.63 \cdot 10^{-34}$ J s, Boltzmann's constant is $1.38 \cdot 10^{-23}$J K^{-1}, and the surface of the sun is 5778 K).